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A correction note to "Posterior contraction rates for level sets" (https://doi.org/10.1214/21-EJS1846)

On page. 2654, in considering the multivariate white noise model, the manuscript says that

"the stochastic differential equation $dY_n(t) = f(t)dt + n^{-1/2}dW(t)$ for $t \in [0,1]^d$, where dW is defined through a multivariate stochastic integral with respect to independent standard Brownian motions $(W_1(t_1), \dots, W_d(t_d))$ and that

$$\int g(t)dW(t) \sim \mathrm{N}(0, \|g\|^2) \text{ for any } g \in L^2([0,1]^d)."$$

There is an error in the definition of W. We actually meant by $W = (W_t)_{t\in[0,1]^d}$ the so-called Brownian sheet process, a mean-zero Gaussian process with covariance kernel

$$\mathbb{E}\left[W_t W_s\right] = \prod_{k=1}^{d} \min\left(t_k, s_k\right),$$

for $t = (t_1, \dots, t_d)$ and $s = (s_1, \dots, s_d)$. This process is also called a multiparameter Gaussian process whose properties have been studied, for instance, in [1], [2] and [3]. In particular, it holds that

$$\int g(t)dW(t) \sim \mathrm{N}(0, \|g\|^2) \text{ for any } g \in L^2([0,1]^d).$$

With the choice of wavelets basis $\{\Psi_{j,k}\}_{j,k}$, we have the equivalent sequence space model given by

$$Y_{j,k} = \theta_{j,k} + \frac{1}{\sqrt{n}}\varepsilon_{j,k}, \quad k \in \{0, \dots, 2^j - 1\}^d, j \geq 0,$$

where the parameters are given by $\theta_{j,k} := \langle f, \Psi_{j,k} \rangle$ and $\varepsilon_{j,k} = \langle W, \Psi_{j,k} \rangle$ are all i.i.d. N(0,1).

References

1. Park, W.J., 1970. A multi-parameter Gaussian process. The Annals of Mathematical Statistics, 41(5), pp.1582-1595.
2. Zimmerman, G.J., 1972. Some sample function properties of the two-parameter Gaussian process. The Annals of Mathematical Statistics, pp.1235-1246.
3. Sottinen, T. and Viitasaari, L., 2015. Fredholm representation of multiparameter Gaussian processes with applications to equivalence in law and series expansions. Modern Stochastics: Theory and Applications, 2(3), pp.287-295.